

**W+jets as a background to top physics: the quest for many jets**S. SCHUMANN<sup>(1)</sup>

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**Summary.** — The latest progress in calculating electroweak gauge boson production in association with QCD jets at hadron colliders is summarized. Particular emphasis is given to the recently completed QCD one-loop calculations of W+3jets and Wb final states. Furthermore recent developments in improving Monte Carlo event generators by means of combining tree-level matrix elements with parton showers is reviewed.

PACS – 12.38.Bx, 12.38.Cy, 13.85.Hd, 13.87.-a.

**1. – Introduction**

Top-physics is at the heart of the Tevatron and LHC physics programme. Since its first observation at the Tevatron in 1995 [1, 2] at lot of effort went into measuring top-quark production cross sections, its mass and quantum numbers. At the LHC top-quark physics can offer a unique window into potential new physics at the TeV scale, see for instance [3].

When considering the semi-leptonic decay of a produced pair of top-quarks the resulting final state is  $l^\pm + E_T^{miss} + \text{jets}$  (where up to two jets might be heavy-flavor tagged). The very same signature is provided by a leptonically decaying  $W$  associated by a corresponding number of QCD jets. To highlight the importance of the  $W + n$ -jets processes as the dominating background to top-pair production, the LHC production cross sections for  $t\bar{t}$ +jets and  $W$ +jets are presented in Fig. 1.

Here only a generic set of cuts, namely  $E_{T,j} > 30$  GeV,  $E_{T,l} > 20$  GeV,  $\Delta R_{j(j,l)} > 0.4$  and  $|\eta_{j,l}| < 2.5$ , on the final-state leptons and jets (including those from the top decays) has been applied. From the left panel we can infer that the probability of producing a  $t\bar{t}$  pair in association with one or more extra jets is quite significant, an observation that is confirmed by the corresponding one-loop calculations [6, 7]. These production rates have to be confronted with the  $W + n$ -jets cross sections, displayed in the right panel. While the inclusive  $W$  rate exceeds the  $t\bar{t}$  cross section by orders of magnitude, when asking for  $\geq 3$  jets the rates become of same size. However, even for  $W + 6$ -jets, background to the semi-leptonic  $t\bar{t} + 2$ -jets process, at leading order (LO) background exceeds the signal.

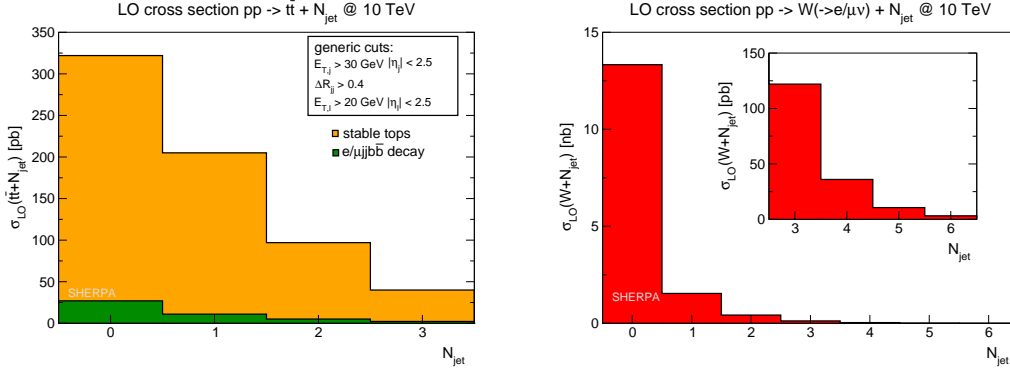


Fig. 1. – The leading-order production cross sections for  $t\bar{t} + n$ -jets (left panel) and  $W + n$ -jets (right panel) in  $pp$  collisions at  $\sqrt{s} = 10$  TeV. For the top signal the inclusive cross sections for stable top quarks and for the semi-leptonic decay channel are shown. For the latter and for the  $W$ +jets background a set of generic jet and lepton cuts have been applied. All cross sections have been calculated using SHERPA [4] employing matrix elements from COMIX [5].

From these simple considerations it is evident that there is a strong demand for having predictions for the  $W + n$ -jets processes accurate at next-to-leading order (NLO) in QCD, reducing inherent scale uncertainties of the theoretical predictions. Furthermore an accurate modelling of this class of high jet-multiplicity processes in Monte Carlo event generators is of major importance for the success of the ambitious LHC top-physics menu.

## 2. – $W+3$ jets at next-to-leading order

Until recently NLO predictions have been available only for final states involving a  $W$  boson and up to two additional jets [8]. Significant progress in the evaluation of virtual matrix elements involving many external legs has enabled two independent groups to eventually calculate  $W + 3$ jets at one-loop accuracy.

In Refs. [9, 10] the leading-color approximation to the full result has been presented. In this calculation the D-dimensional generalized unitarity method as described in Ref. [11] is used to evaluate the loop amplitudes. The actual calculation is performed in the framework of the MCFM code [12]. The authors of Refs. [9, 10] proposed a prescription called "leading color adjustment" that allows them to provide a sensible approximation to the full-color NLO result. In essence they rescale the leading color one-loop result by a constant factor defined to be the ratio of the LO full color cross section over its leading-color approximation.

In Refs. [13, 14] the first complete NLO calculation of  $W + 3$ jets has been presented. This calculation includes all partonic subprocesses and is exact in the treatment of color. For the one-loop matrix elements the program BLACKHAT [15] is used that is based on unitarity methods [16, 17, 18]. For the generation of the real-emission matrix elements, the Catani–Seymour dipole subtraction terms [19], as well as all phase-space integrations the Monte Carlo generator SHERPA [4, 20] is used.

From NLO calculations we can expect a reduced dependence on the unphysical renormalization and factorization scales. However, they still exhibit a scale dependence.

# of jets	CDF	LO	NLO
1	$53.5 \pm 5.6$	$41.40(0.02)^{+7.59}_{-5.94}$	$57.83(0.12)^{+4.36}_{-4.00}$
2	$6.8 \pm 1.1$	$6.159(0.004)^{+2.41}_{-1.58}$	$7.62(0.04)^{+0.62}_{-0.86}$
3	$0.84 \pm 0.24$	$0.796(0.001)^{+0.488}_{-0.276}$	<b><math>0.882(0.005)^{+0.057}_{-0.138}</math></b>

TABLE I. – Total cross sections in pb with scale dependence for  $W + n$ -jets at Tevatron compared to data from CDF [21]. Numbers taken from Ref. [14], a complete description of the calculational setup and the cuts used can be found therein.

In Tab. I the theoretical prediction for the  $W + 1, 2, 3$ -jets cross sections calculated at LO and NLO are compared to a measurement by CDF [21]. The inherent scale uncertainties are indeed significantly reduced for the one-loop results. The newly obtained  $W + 3$ jets NLO result is in perfect agreement with the data. The predicted scale uncertainty for  $W^\pm + 3$ jets production at the LHC is also largely reduced at NLO. Considering  $pp$  collisions at 14 TeV and  $E_T^{jet} > 30$  GeV, Ref. [14] quotes

$$\begin{aligned} \sigma_{W^-+3\text{jets}}^{\text{LO}} &= 22.28(0.04)^{+7.80}_{-5.34} \text{ pb} & \text{vs.} & \quad \sigma_{W^-+3\text{jets}}^{\text{NLO}} = 27.52(0.14)^{+1.34}_{-2.81} \text{ pb}, \\ \sigma_{W^++3\text{jets}}^{\text{LO}} &= 34.75(0.05)^{+12.06}_{-8.31} \text{ pb} & \text{vs.} & \quad \sigma_{W^++3\text{jets}}^{\text{NLO}} = 41.47(0.27)^{+2.81}_{-3.50} \text{ pb}. \end{aligned}$$

Besides a reduced scale dependence of the total  $W + 3$ jets cross section the NLO calculation exhibits largely narrowed uncertainty bands for differential distributions. This is exemplified in Fig. 2, where the transverse-momentum distribution of the third-hardest jet at Tevatron and LHC energies is shown. However, care has to be taken which central scale is actually used in this intrinsic multi-scale problem. As pointed out in Refs. [10, 14] a choice like the bosons transverse momentum,  $E_T^W$ , can yield unphysical results for certain distributions, originating from large kinematic logarithms. A seemingly more appropriate choice is the total partonic transverse energy,  $\hat{H}_T$ .

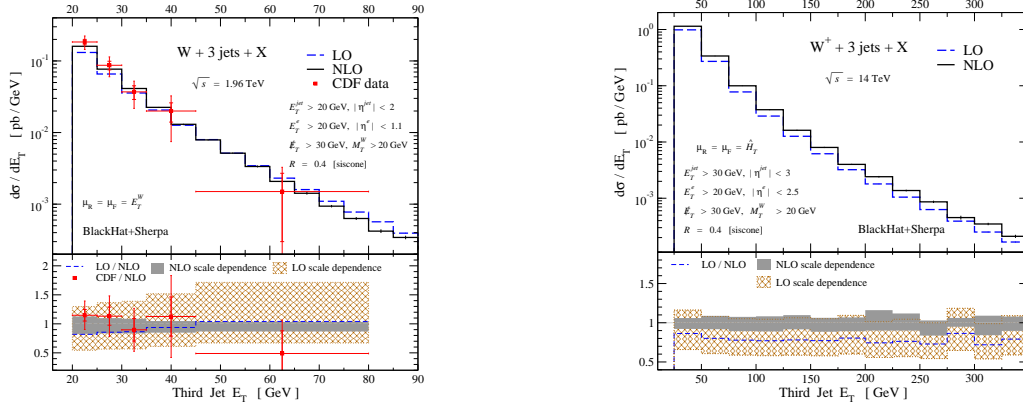


Fig. 2. – Transverse-momentum distribution of the third-hardest jet in  $W + 3$ jets events at the Tevatron, compared to data from CDF [21] (left panel) and the LHC (right panel) at leading- and next-to-leading order. Figures taken from [14].

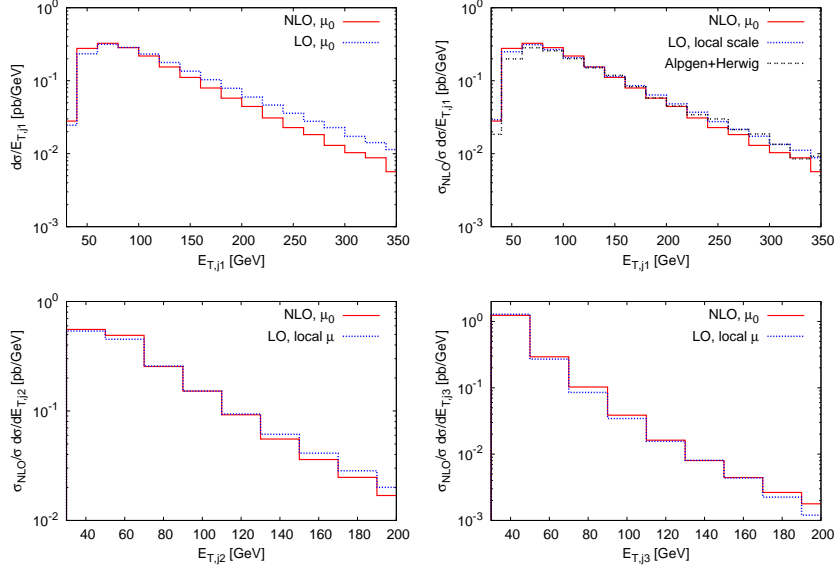


Fig. 3. – Jet transverse-momentum distributions in  $W^+ + 3\text{jets}$  production at a 10 TeV LHC. The default scale is chosen to be  $\mu_0 = \mu_R = \mu_F = \sqrt{p_{T,W}^2 + m_W^2}$ . The local scale choice corresponds to taking each  $\alpha_S$  factor in the LO calculation at its respective  $k_T$  splitting scale. For details on the calculational setup see [22].

Ref. [22] discussed the possibility to accommodate shape differences between NLO and LO results by appropriately choosing scales for the strong coupling factors in the latter. In particular a local scale choice was investigated where each  $\alpha_S$  factor is evaluated at a reconstructed  $k_T$  splitting scale. In Fig. 3 a comparison between the default scale  $\mu_0 = \sqrt{p_{T,W}^2 + m_W^2}$  and the local prescription for the transverse-momentum distribution of the three hardest jets is shown. The local scale scheme is in much better agreement with the NLO shapes. This approach of local  $\alpha_S$  factors is commonly used in parton shower Monte Carlos and in particular in calculations that combine tree-level multi-parton matrix elements with showers [23]. The explicit comparison for  $W + 3\text{jets}$  final states at NLO confirms observations made in Refs. [24, 25] for  $W + 1, 2\text{jets}$  production and re-affirms the predictive power of the matrix element parton shower approach. For a further study along these lines see [26].

### 3. – $W$ +heavy flavors at next-to-leading order

Concerning backgrounds to top-quark production special attention has to be given to  $W$ +jets final states with one or two jets being  $b$ -tagged. Using massive partons in the theoretical calculation removes corresponding soft and collinear singularities as they are regulated by the finite quark mass, however at the price of the fixed-order calculation being more difficult. At present the NLO corrections for  $Wb\bar{b}$ , with massive  $b$ -quarks, are known [27]. When applying cuts that suppress contributions from the threshold region  $m_{b\bar{b}} \approx 2m_b$  the actual difference between the fully massive calculation and the limit  $m_b = 0$  is typically less than 10% [27].

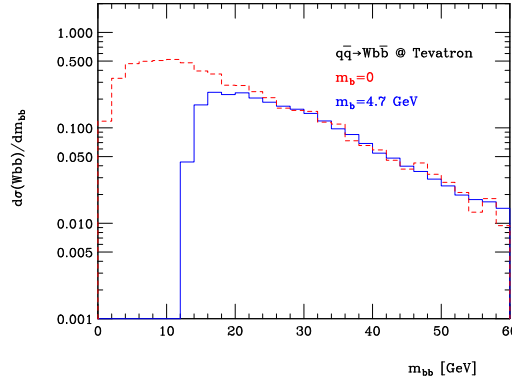


Fig. 4. – LO differential cross section for  $Wb$  production at Tevatron as a function of the  $b\bar{b}$  invariant mass. Only one b-quark is found inside the fiducial volume here. Figure taken from [28], further details can be found therein.

However, when sensitive to the threshold region or in case that just one heavy quark is tagged the massless approximation is not applicable. In the latter case the unobserved heavy jet must be integrated over the whole phase space thus introducing reference to the b-quark mass, cf. Fig. 4. One way out is to use heavy-quark parton distribution functions – the so-called variable flavor scheme (VFS) that has the additional advantage to re-sum large logarithms of the type  $\ln(m_W/m_b)$  to all orders.

In Ref. [28] a full NLO calculation of producing a  $W$  boson in association with just a single b-jet has been presented. This calculation consistently combines the massive  $Wb\bar{b}$  calculation of [27] with the VFS computation of  $Wbj$  [29]. This calculation is an important ingredient when comparing the recent CDF measurement of the  $W$  associated b-jet cross section [30] with the NLO QCD calculation

$$\sigma_{b-jets}^{\text{CDF}} \times \mathcal{B}(W \rightarrow l\nu) = 2.74_{-0.50}^{+0.50} \text{ pb} \quad \text{vs.} \quad \sigma_{b-jets}^{\text{NLO}} \times \mathcal{B}(W \rightarrow l\nu) = 1.22_{-0.14}^{+0.14} \text{ pb}.$$

There is obviously tension between experiment and the theoretical results from NLO QCD as well as Monte Carlo predictions relying on matrix-element parton-shower merging [30]. The source of this disagreement is still under study - but might be assigned to the scale choice in the calculations [31].

#### 4. – Monte Carlo event generators

In cases we lack a full NLO calculation (e.g.  $W + \geq 4\text{jets}$ ) or observables are sensitive to multiple-parton emission and hadronization effects, theoretical predictions rely on the ability of multi-purpose Monte Carlo generators such as PYTHIA [32], HERWIG [33] or SHERPA [4, 34] to account for the underlying physics. Over the past decade enormous efforts went into improving these calculations by consistently incorporating multi-leg tree-level matrix elements into parton-shower simulations in the spirit of [23, 35, 36]. For an overview of available approaches and an extensive comparison for  $W$ +jets production at Tevatron and LHC see Ref. [38]. Essentially two major problems have to be addressed by each tree-level merging algorithm:

- How to attach a parton shower to a multi-leg tree-level matrix-element calculation without spoiling the logarithmic accuracy of the underlying QCD resummation?
- How to avoid potential double- or under counting of phase-space configurations present in the parton shower and corresponding matrix-element calculations?

To accommodate these conditions in a generic tree-level merging algorithm

- multi-parton matrix elements get regularized through a suitably defined jet measure (e.g. a critical  $k_T$ - or cone-like distance);
- appropriate starting conditions for the initial- and final-state parton shower have to be determined and certain (hard) shower emissions need to be vetoed.

In particular the second item is subject to certain approximations in the various schemes. An important concept to overcome those approximations is a so-called *truncated shower*, first proposed in Ref. [39]. The underlying observation is that due to a mismatch of the jet-measure, used to slice the emission phase space, and the actual shower-evolution variable the radiation pattern of soft/large-angle emissions can be distorted.

In Ref. [40] such a truncated shower was implemented for the first time. The implementation relies on the shower algorithm based on Catani–Seymour dipole factorization [41] and combines it with the matrix-element generators available inside the SHERPA framework. The method has successfully been applied to jet production in  $e^+e^-$  collisions, the Drell-Yan process [40], prompt-photon production [42] and deep-inelastic scattering [43]. As of version 1.2 it constitutes the default method for combining matrix elements with parton showers in the SHERPA generator. The new merging approach yields a largely reduced dependence on the intrinsic merging parameters compared to the previous CKKW implementation in SHERPA and other merging algorithms [38]. This is illustrated by the systematics studies for  $Z^0/\gamma^*$ +jets production at Tevatron presented in Figs. 5 and 6. The first figure presents a comparison of the jet multiplicity and leading-jet  $p_T$  distribution while in the latter the variation of the  $k_T$  differential jet rates  $d_{1\rightarrow 0}$  and  $d_{2\rightarrow 1}$  for three different values of the slicing measure are presented.

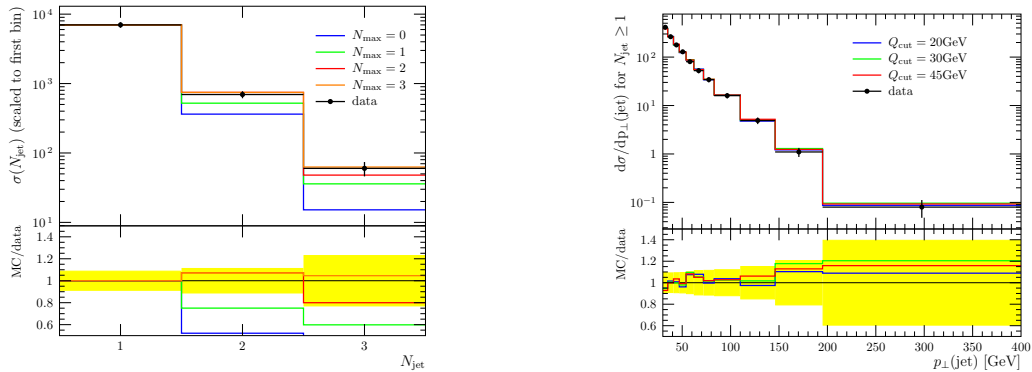


Fig. 5. – Jet multiplicity (left panel) and the leading jet  $p_T$  spectrum (right panel) in inclusive  $Z^0/\gamma^*$ +jets production compared to data from CDF [37]. Figures taken from Ref. [40]

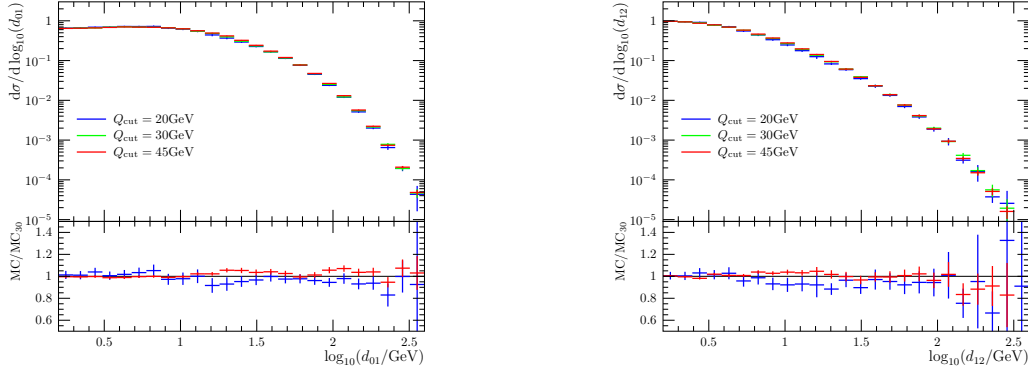


Fig. 6. – Differential jet rates  $d_{01}$  (left panel) and  $d_{12}$  (right panel) for the CDF Run II  $k_T$ -algorithm [44]. Displayed are the predictions for three different values of the merging cut. Figures taken from Ref. [40]

## 5. – Conclusions and Outlook

Understanding the process of electroweak gauge boson production in association with QCD jets is crucial for the success of the top-physics programme both at the Tevatron and even more so at the LHC. In the last few years there has been enormous progress in the calculation of one-loop corrections to multi-parton final states. As a result the processes  $W + 3\text{jets}$  and  $Z^0/\gamma^* + 3\text{jets}$  are meanwhile known at next-to-leading order in QCD. In fact Ref. [45] already reports on first steps towards the calculation of  $W + 4\text{jets}$  at the one-loop level using the BLACKHAT+SHERPA package. At this conference M. Worek reported on the HELAC-NLO package, that has proven to be capable of doing calculations of this complexity as well and next-to-leading order calculations for  $Wb\bar{b} + \leq 3\text{jets}$  now seem to be feasible.

Concerning the simulation of  $W$ +jets with Monte Carlo event generators a high level of sophistication has been reached. The approach of combining multi-leg tree level matrix elements with parton showers has become a widely used standard that delivers results in good agreement with data from Tevatron and exact higher-order calculations. One important future direction will be to precisely understand how these methods can be generalized to allow for the inclusion of one-loop matrix elements. First proposals in this direction have been made and implemented already, cf. Refs. [46, 47] and P. Nason's contribution to these proceedings. A novel procedure how to combine next-to-leading calculations of different final-state multiplicity, though not facing the problem of attaching parton showers, has been presented in Ref. [48].

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The author would like to thank the organizers for the invitation and in particular Fabio Maltoni. Financial support from BMBF is gratefully acknowledged.

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